Clarification / correction:

C_n & W_n are defined for $n \geq 3$

K_n defined for $n \geq 1$

K_1 :  K_2 : 

Recursion

recursive function defn has 2 parts:

1. base case(s)
2. recursive formula

Fibonacci numbers:

$F_0 = 0$
$F_1 = 1$ } base cases

$F_i = F_{i-1} + F_{i-2} \quad \forall i \geq 2$

$F_5? = F_4 + F_3$

$(F_3 + F_2) + (F_2 + F_1)$

$(F_2 + F_1) + (F_1 + F_0) + (F_1 + F_0) + F_1$

$F_1 + F_0 + F_1 + F_1 + F_0 + F_1 + F_0 + F_1$

$1 + 0 + 1 + 1 + 0 + 1 + 0 + 1 = 5$

unrolling, since my input = 5 I completed unrolling up to base case

0, 1, 1, 2, 3, 5 "normal way"

↓ 0th ↓ 5th

unrolling: technique to find closed form of a recursive function
not a proof technique.
induction to prove it.

example: $T: \mathbb{N} \to \mathbb{Z}$

$$\boxed{T(1) = 1}$$

$$T(n) = 2T(n-1) + 3$$

level

$k=1$ $\quad T(n) = 2\,T(n-1) + 3$

$k=2$ $\quad T(n) = 2\left(2T(n-2) + 3\right) + 3$

$k=3$ $\quad T(n) = 2\left(2\left(2T(n-3)+3\right)+3\right)+3$

$k=3$ $\quad T(n) = 2^3 T(n-3) + 2^2 \cdot 3 + 2^1 \cdot 3 + 2^0 \cdot 3$

k? $\quad T(n) = 2^k \underbrace{T(n-k)}_{= T(1)} + \sum_{i=0}^{k-1} 2^i \cdot 3$ $\quad$ 1

$n - k = 1 \rightarrow k = n-1$ $\quad$ 2

value of k at base case of 1

$$2^{n-1} T(n-(n-1)) + \sum_{i=0}^{n-2} 2^i \cdot 3$$

$$2^{n-1} \cdot \underbrace{T(1)}_{1} + 3 \sum_{i=0}^{n-2} 2^i$$

$$2^{n-1} + 3 \sum_{i=0}^{n-2} 2^i$$

$$2^{n-1} + 3\left(2^{n-1} - 1\right)$$

$$2^{n-1} + 3 \cdot 2^{n-1} - 3$$

$$4 \cdot 2^{n-1} - 3$$

$$2^2 \cdot 2^{n-1} - 3$$

$$\boxed{2^{n+1} - 3}$$ closed form

example: $S(1) = C$

$S(n) = 2S(n/2) + n \quad \forall n \geq 2$

$S(6) = 2S(3) + 6$

$S(n/2) = 2S(n/4) + n/2$ ✲ don't forget

level

$k=1 \quad S(n) = 2S(n/2) + n$

$k=2 \quad S(n) = 2(2S(n/4) + n/2) + n$

$k=3 \quad S(n) = 2(2(2S(n/8) + n/4) + n/2) + n$

$k \quad S(n) = 2^k S(n/2^k) + kn$

$k?$ $\quad S(1)$

$n/2^k = 1$

$n = 2^k$

$k = \log_2 n$

$S(n) = 2^{\log_2 n} \cdot S(n/2^{\log_2 n}) + (\log_2 n)(n)$

$S(n) = n \cdot S(1) + n\log_2 n$

$\boxed{S(n) = n \cdot C + n\log_2 n}$ closed form

✲ Do 12.2 bc instead of 12.2 ab